Modeling the Evolution of the Internet Topology: A Multi-Level Evaluation Framework

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***Abstract*—The topology of a network (connectivity of autonomous systems (ASes) or routers) has significant implications on the design of protocols and applications, and on the placement of services and data centers. Researchers and practitioners alike are in constant need of realistic topologies for their simulation, emulation, and testbed experiments. In this work, we propose a multi-level framework for analyzing Internet topologies and their evolution. This multi-level approach includes novel metrics, evaluation strategies, and techniques for automatically learning a representative set of graph metrics. We apply our framework to analyze topologies from two recent topology generators, Orbis and WIT, according to how well they match their advertised objectives. The generated topologies are compared to a set of benchmark datasets that approximate different views of the Internet at the data (trace-route measurements), control (BGP), and management (WHOIS) planes. Our results demonstrate key limitations of popular generators, and show that the recent Internet clustering coefficient and average distance are not time-invariant as assumed by many models. Additionally, we propose a taxonomy of topology generators, and identify challenges in topology modeling.**

I. Introduction

Accurate representation of network topologies plays a critical role in designing protocols [28], predicting performance [21], and understanding robustness and scalability of the future Internet [22], [17], [29]. Many topology generators are known to capture key static properties of the Internet, but may not capture Internet evolution. The most recent Internet topology generators [25], [35], [33], [24], [4] aim at generating representative topologies of different sizes (number of nodes); however, understanding and modeling the driving forces behind Internet evolution remains a significant challenge.

Looking back on Internet topology modeling research over the past 10 years, Roughan *et al.* [30] point out that the majority of prior work makes simplifications that lead to misleading findings and ill-conceived ideas about the Internet. In this work, we examine this prior research on topology modeling quantitatively. We propose a taxonomy for topology generators, analyze the claims made by each, and introduce a multi-level framework for evaluation. In doing this, we seek to answer the following questions: **(1)** As the Internet evolves over time, are its graph properties constant? **(2)** How well do existing topology generators model the static and evolutionary properties of the Internet? **(3)** What type of topology generator is more appropriate for static topologies versus evolving topologies?

Several previous topology generation studies examined the average degree, average path length, and the clustering coefficient and showed them to remain constant as the Internet topology evolves, *e.g.,* [34], [25]. These findings are based on RouteViews datasets from the 1990s–2006 time frame. We will show different evolutionary characteristics in more recent RouteViews data (Fig. 1). Further, several generators aim to maintain "any arbitrary metric" of a generated topology constant as the number of nodes increase, *e.g.,* [25], [26]. However, there is evidence of the Internet topology transitioning from hierarchal to a flat topological structure [13], [12], [23]. These observations of a significant change in the evolutionary characteristics of the Internet are consistent with our findings, and explain why topology generators that assumed a prior structure or process based on previous patterns fail to predict recent evolution of the Internet topology.

We compare generated topologies to real and synthetic datasets based on a multi-level approach that includes novel metrics, evaluation strategies, and techniques for automatically learning a representative set of graph metrics. In contrast to prior work that assumed that a few metrics are sufficient and necessary to evaluate generated topologies, we find that the metrics previously studied are not enough to show the superiority of a generator over another. There are often other factors to consider. For instance, we found that under certain conditions, Orbis generates topologies that are approximately isomorphic to the initial topology used as input. Therefore, any graph metric used to evaluate these topologies will match almost exactly. As a result, it may be important to require some variance between the graph used as input and the generated graphs.

The contributions of this paper can be summarized as follows. *First,* we propose a multi-level framework for evaluating topologies and their evolution. This multi-level framework includes graph, node, and link metrics. *Second,* we study the clustering coefficient and average distance in recent Internet AS topology data, and demonstrate that they are not time-invariant as assumed by earlier work. *Third,* we apply our multi-level framework to evaluate two recent popular topology generators, Orbis [25] and WIT [35], according to whether they produce graphs that match their advertised objectives. The topology generators are compared to a variety of datasets that approximate different views of the Internet at the *data* (traceroute measurements [8]), *control* (BGP [2]), and *management* (WHOIS [1]) planes. We also give a few results using the RocketFuel [31] and the HOT [24] router-level topologies.

We organize the paper as follows. Section II proposes a taxonomy for topology generators. We also discuss in detail two recent topology generators, Orbis and WIT. In section III, we propose a framework for analyzing network properties based on matrix factorization techniques that range across three-levels of structural granularity. Section IV describes the datasets and process used to compare real and synthetic topologies. In section V, we evaluate the topology models using our multi-level approach. We conclude with a summary of our main findings.

II. Taxonomy and Challenges

We introduce a taxonomy of prominent topology generators in Table I. This taxonomy is based on the main principle used in generating topologies (random-walk, optimization, preferential attachment, geometry), the type (parametric, nonparametric), and the topology (AS, router-level (RL)) constructed by each.

Parametric generators assume a particular functional form or mechanism, whereas nonparametric models are data-driven and make fewer assumptions about the functional form. A generator may estimate parameters from data or simply assume an underlying mechanism. If an underlying mechanism or principle is used in a topology generator (*e.g.,* WIT), the generator is tailored to a specific type of topology such as the Internet AS topology, whereas topology generators that perform parameter estimation from data can typically generate many types of networks (*e.g.,* social networks, biological networks).

Models based on *preferential attachment* [3], [5], [27], [20], [7], [36] may be too restrictive to model the Internet evolution, since the decision to link to another node is based on degrees of nodes, giving high importance to highly connected nodes. Generators based on random-walks can suffer from similar problems. Optimization-based generators [10], [14] consider solving the *optimization* problem between the benefit and improved connectivity of the network. However, economic considerations of the connectivity are not typically considered in the optimization. GT-ITM [9] is one of the earliest generators and is primarily based on the *hierarchical* nature of the Internet, which appears to be changing [13], [12], [23]. Tangmunarunkit *et al.* [32] find that degree-based models match a set of metrics better than these early hierarchical models. Though recent generators accurately match the power-law distribution, they fail to capture certain network metrics summarized in Table II.

TABLE I
Taxonomy of topology generators

Technique	**Generators**	**Model Type**	**Topology**
Random-walks	**WIT** [35]	Parametric	AS
	RSurfer [6]	Parametric	N/A
Optimization	**Orbis** [25]	Nonparametric	AS & RL
	HOT [24]	Parametric	RL
	Mod. HOT [10]	Parametric	AS
Preferential Attachment	**AB** [3]	Parametric	N/A
	BRITE [27]	Nonparametric	AS & RL
	Inet [20]	Parametric	AS
	GLP [7]	Parametric	AS
Geometry	**SWT** [21]	Parametric	AS & RL
	GT-ITM [9]	Parametric	AS & RL

Below, we formally describe two recent topology generators, Orbis and WIT, which claim to accurately model the Internet and which are the primary focus of the remainder of this paper.

A. Orbis: From Degree to Topology

Orbis uses a series of metrics based on degree correlations that monotonically capture more global structures [25]. An initial topology is given as input and randomly rewired such that the dK-distribution of the input topology is preserved. The topologies are rescaled by simply stretching the target distribution and preserving it under random rewiring. The chosen value of d must be small in practice due to the increase in complexity and the shrinking of the graph space. Therefore, Orbis is limited to preserving only local characteristics. As d increases, the space of possible graphs that can be generated exponentially shrinks, yielding topologies that are only slightly different from the input training topology. Of course, if the input graph varies only slightly from the generated graph, then both topologies exhibit nearly identical characteristics, and the generated graph is not too useful. As we will show, constructing graphs with $d = \{1, 2, 3\}$ captures a few related local metrics, but fails to capture global characteristics. Furthermore, as the generated graphs are rescaled, the accuracy of capturing the local metrics depends on the rescaling technique, and thus becomes increasingly inaccurate as the size of the topology increases.

B. WIT: From Random Walks to Topology

The WIT model uses a simple multiplicative stochastic process, $u_i(t) = \lambda_i(t)\ u_i(t-1)$, capturing the "wealth" of ISPs over time [34], [35]. The wealth (or weight) for each ISP is used to add or remove links based on a given threshold.

More formally, at each iteration (or time) a nodes wealth is updated. If the updated node weight exceeds a given threshold $w_i(t) - z_i(t) > C + T$ then a link is added by randomly walking l-steps until an arbitrary node z is reached and a link is placed between the nodes. In the above threshold, the $w_i(t)$ is the normalized wealth for node i and $z_i(t) = C \cdot d_i(t)$ (depends on the current degree of node i and the expected link cost $C = w_0 \cdot c$ where w_0 and c are only some of the input parameters). Furthermore, T must carefully be chosen to avoid an oscillating (degenerate) case where a link is added and at the next time deleted, indefinitely. Similarly, if the node weight is below a threshold $w_i(t) - z_i(t) > -T$, then the node randomly chooses one of its links to remove. A new node x is added and a link is initially created by randomly selecting a node y and linking to that node. Additionally, a second link is added by randomly walking l-steps as described previously.

C. Challenges in Topology Generation

Topology measurement and modeling remains challenging [30]. As discussed above, some models like WIT do not estimate parameters using an input topology, but rather rely on the accuracy of assumptions made about the underlying growth mechanism and the ability of this mechanism to accurately match the forces driving the evolution of the Internet topology. Selecting the parameters for WIT requires detailed knowledge of the model and the behavior of the Internet over time, which is not very well-understood. The lack of parameter estimation makes WIT difficult to use in simulations or other practical applications. The initial evaluation of WIT in [35] uses the optimal parameters for RouteViews, but provides little intuition for selecting these parameters for a given AS topology.

TABLE II
GRAPH METRICS AND RELATED NETWORK CHARACTERISTICS

	Measure	Importance in Computer Networks
LOCAL	Degree	Fault tolerance, local robustness
	Assortativity	
	Clustering coefficient	Path diversity, fault tolerance, local robustness
GLOBAL	Distance	Scalability, performance, protocol design
	Betweenness	Traffic engineering, potential congestion points
	Eigenvector	Network robustness, performance, clusters/hierarchy, traffic engineering

We can thus classify topology generators as those based on preserving the properties of an *input topology* or those based on a *mechanism or knowledge of the underlying process* being modeled. Consider the space of all graphs that preserve a set of metrics and suppose we select an arbitrary graph from this space. This graph may preserve the set of metrics, but not capture the qualitative characteristics of the system (*i.e.,* AS topology). Therefore, a graph that preserves a set of metrics is not guaranteed to capture the important characteristics of the system under investigation. From this perspective, the WIT model attempts to model the evolution of the AS topology directly, whereas Orbis generates topologies that preserve a set of metrics. Thus, the WIT model would fail when the underlying process used in the formation and growth of the Internet changes or if the model fails to capture the underlying process, whereas Orbis would fail if the set of characteristics are incomplete with respect to the characteristics implied by the actual AS topology.

III. A Multi-Level Framework

At the heart of our topology and evolution analysis framework are multi-level evaluation criteria based on three general classes of metrics: graph, node, and link metrics. Graph metrics (*i.e.,* scalar metrics) compute a single value for the entire network graph, whereas node and link metrics compute a value for each individual node or link, respectively. The three evaluation granularities form an explicit ordering, $link\ metrics \geq node\ metrics \geq graph\ metrics$, according to the difficulty of preserving a property from that level. A topology generator that consistently preserves graph metrics is said to be validated at this level (*i.e.,* the least constrained evaluation granularity). Most previous work uses only graph metrics to evaluate the effectiveness of topology generators; however, these measures can be misleading or trivially satisfied. The node and link metrics capture intrinsic graph structure and connectivity better than single point statistics.

The three classes of metrics can be further categorized as macro metrics (graph) or micro metrics (node and link). Evaluating topology generators at the micro-level is clearly more accurate than evaluating them at the macro-level. Further, within each class of metrics, topologies can be evaluated at different *topological granularities* (*e.g.,* global $\geq$ clusters/communities $\geq$ local). *Temporal granularity* (*e.g.,* days, months, years) is also critical for studying evolution.

As a basis for our multi-level approach, we derive *graph*, *node*, and *link* metrics using matrix factorizations including singular-value decomposition, spectral decomposition (singular-value decomposition and spectral decomposition are equivalent for a square symmetric positive definite matrix), and non-negative matrix factorization. The importance of these matrix factorization techniques and their characteristics pertaining to the AS-level and router-level topologies have only received limited attention. The majority of previous work uses the spectrum of the Laplacian matrix (which indicates connected components and sparse cuts) and ignores the adjacency matrix (which can indicate properties such as the number of paths). Nevertheless, matrix factorization-based characteristics have been described by some as critically important [26], [16], [11], yielding tight bounds for (1) distance-related characteristics, (2) clustering-related characteristics, and (3) graph resilience under node/link removal. Most graphs with large eigenvalues exhibit small diameters, expand faster, and are more robust with respect to link or node removal. The eigenvectors also cluster tightly connected or similar nodes, whereas the large eigenvalues may imply more node and link-disjoint paths. In addition to capturing a significant amount of global information about the resulting topology, the eigenvectors identify local characteristics such as degree-patterns and clustering of various sizes. Thus, metrics based on matrix factorizations can estimate the preservation of most important network characteristics required by researchers investigating various networking problems. Table II gives for a summary of the metrics and the corresponding important network characteristics.

Further, previous work evaluates topologies by comparing the values of individual metrics, whereas in this work we use KL-divergence to compare topologies using the combined set of metrics. This allows us to more accurately and systematically evaluate generators and their topologies using a single quantitative measure. In particular, we measure the divergence of an entire set of graph metrics over time from each generator where the distribution of graph metrics from the RouteViews topologies is used as the true distribution (ground-truth). We also use an approach that automatically learns a set of graph features for each generator and evaluates them using the KL-divergence.

A. Graph Metrics

In addition to the traditional macro metrics (*e.g.,* average degree k, assortativity coefficient r, average clustering $\bar{c}$, average distance $\bar{d}$), we propose using the largest singular value (λ_1), network conductance ($\lambda_1 - \lambda_2$), rank (ρ), and trace (τ). The rank of a matrix is the number of linearly independent rows or columns and the trace is the sum of its eigenvalues, which is invariant with respect to a change of basis. The difference between the two largest eigenvalues denotes the *network conductance* which is also known by some as the performance of a network [26]. Each of these scalar values are derived from the eigenvectors and eigenvalues (or, if appropriate, the singular-vectors or singular-values) of the graphs adjacency matrix. Additionally, we use the decay centrality $\bar{\eta}$ which seeks to more accurately capture average distance based measures [19].

B. Node Metrics

To analyze the local and global node-level properties of networks, we primarily use measures derived from matrix factorizations, including:

Network Values. Plot of the eigenvector components (indicators of network value) corresponding to the largest eigenvalue.

Scree Plot. Plot of the k largest eigenvalues (or singular-values) versus their normalized rank using the log-scale.

K-walks: A Class of Local and Global Measures. We propose using a simple class of metrics, denoted as *k-walks*, capable of measuring both local and global properties of graphs by adjusting a single parameter. A *k-walk* of a vertex u is the number of walks of length k rooted at u.

The number of walks from node u to node v in a graph G with length k is $(A^k)_{uv}$. The k-walk measure of a graph adjacency matrix is given by, $\sigma_k(\mathcal{A}) = \mathcal{A}^k\mathrm{x}$, where x is the unit vector. If $k \to \infty$ then we have the principal eigenvector and the other extreme where $k = 1$ results in the degree distribution. Intermediate values $1 \leq k \leq \infty$ give other properties of the graphs going from the most local property of degree to the most global property of the principal eigenvector. This metric provides a formal way to bound the similarity of two graphs with respect to k.

C. Link Metrics and Topology Visualizations

Link metrics lie at the finest evaluation granularity. First, we apply a technique to order the nodes with respect to the magnitude of their coordinates along the principal direction. This procedure reveals significant link structures, connectivity patterns, and block structures/clustering. Second, we analyze the network characteristics more accurately by computing the closest k-approximation of the topology resulting in the weighting, suppression, or creation of links. Similar techniques have been used in information retrieval and various fields that require a low-dimensional representation that preserves the most significant information with minimum loss. Both methods allow us to identify the most significant properties preserved in the resulting topology.

D. Learning Graph Features over Time

In contrast to only selecting a set of graph metrics, we automatically learn a representative set of graph features. For this, we use ReFex [18]. The technique generates simple degree and "egonet" features, and then recursively discovers additional features by applying aggregation. The set of graph measures are pruned by examining correlations. The resulting graph features are shown to be representative of the topology and minimal. We use this technique for analyzing and evaluating graphs *over time*. For a time series of topologies, the technique usually learns 25–40 graph features, depending on the evolution and structure of the topologies.

The set of discovered graph features provides an additional basis for evaluating graphs and their generators. In particular, we measure the KL-divergence of the generators using the set of learned features over time. We also use these features to model behavior (*i.e.,* cluster the features into groups) and use this information to distinguish among generators.

IV. Methodology

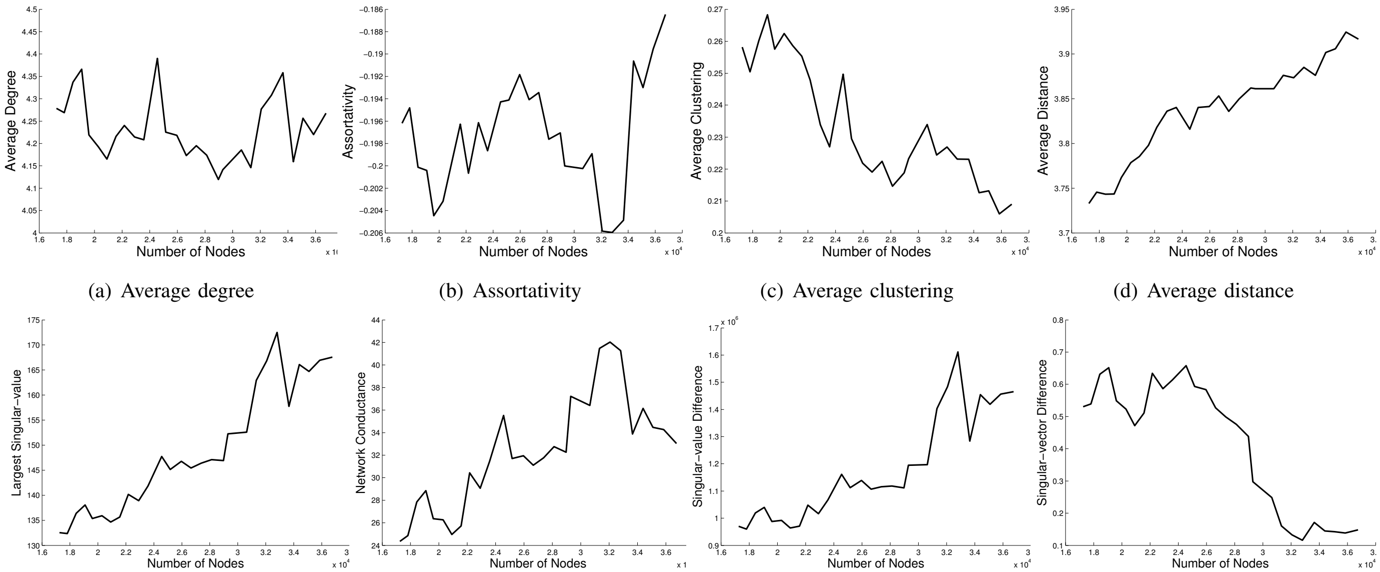
A. Datasets

We compare graphs from topology generators to AS topologies based on the Skitter traceroute [8], RouteViews' BGP tables [2], and RIPE's WHOIS [1] datasets. These are the same datasets used by Mahadevan *et al.* in [25] and all except RouteViews data were obtained from the authors' web site. In addition to these AS-level topologies, we compare to the HOT [24] and RocketFuel [31] router-level topologies. The HOT topologies were also obtained from the web site of Mahadevan *et al.* The RocketFuel topologies were obtained from RocketFuel project site.

To study the most recent evolutionary characteristics of the Internet, we obtained time series of BGP routing tables (in Cisco and Zebra format) from the Oregon RouteViews project [2]. We estimated the AS-level topology by taking the union of all AS-paths in the routing tables as performed by Gao [15]. We extracted AS-level subgraphs for the years 2004 to 2011 from the last 25 BGP tables of March, June, September, and December of the corresponding year (except for 2011 when we consider all tables from January 1 to April 24, 2011). Unlike [12], we do not distinguish customer-provider and peering links, and simply follow the approach taken in WIT evaluation [34], [35].

B. Evaluating Topology Generators

Let $\mathcal{P}$ denote the process (e.g., Orbis, WIT, or the actual Internet) used to generate a set of graphs $\mathcal{G}$ of any size. Further, let G_n be a generated graph of size n nodes from



(a) Average degree (b) Assortativity (c) Average clustering (d) Average distance

(e) Largest singular value (f) Network conductance (g) Singular-value difference (h) Singular-vector difference

Fig. 1. Evolutionary characteristics of the AS topology as observed from RouteViews (2004 to 2011). The majority of previous models assume the Internet topology (and metrics applied to it) are time-invariant. However, as shown above, we find evidence of a recent transition in the topological structure, most notably seen from 2007-present (28K and beyond) which is consistent with [23]. The x-axis is the # of nodes (see Table III for the corresponding years).

the distribution of graphs from a generator denoted $P(\mathcal{G}|\mathcal{P})$. The set of graph metrics (degree, clustering coefficient,...) computed over that graph is denoted as a function $\mathcal{M}(\cdot)$. The evaluation objectives of topology generators are formally defined as,

1) Given a graph $G_n^\star$ of size n, generate a graph of the same size G_n such that $\mathcal{M}(G_n) \approx \mathcal{M}(G_n^\star)$
2) Given a graph $G_n^\star$ of size n, generate a graph G_m of size m where $m \geq n$ such that $\mathcal{M}(G_m) \approx \mathcal{M}(G_n^\star)$
3) Given an ordered sequence of graphs $G_t^\star$ for $t = 1, 2, ..., m$, generate a corresponding sequence of graphs G_t for $t = 1, 2, ..., m$ such that each G_t is the same size as $G_t^\star$ and $\mathcal{M}(G_t) \approx \mathcal{M}(G_t^\star)$

where $\mathcal{M}(G_n) \approx \mathcal{M}(G_n^\star)$ represents the fact that the set of metrics applied to each graph (and the corresponding distributions) are approximately equal. The evaluation objectives 1 and 2 are for Orbis [25] while 3 is for WIT [35]. 1 is a relaxed version of WIT's objective since it considers a graph at a single time point. Further, the second evaluation objective assumes properties of the graph at a smaller size remain unchanged (time-invariant) as the graph grows larger.

WIT does not estimate the parameters of a dataset and thus can be evaluated directly with any AS topology such as RouteViews, Skitter, and WHOIS. For Orbis, we estimate the parameters of each dataset and generate the corresponding topology. To evaluate Orbis over time (rescaling), we use the first snapshot (of RouteViews) and apply the rescaling algorithm to generate a sequence of evolved topologies. This evaluation strategy allows us to evaluate the objectives of each generator separately using the benchmark topologies. We use the optimal parameters for WIT given in [35]; for Orbis, we follow the methodology described in [25] and use the implementation provided by the authors.

V. Evaluation

As a case study, we apply our multi-level framework to evaluate each generator according to whether they produce graphs that match their advertised claims and objectives. The topology generators are evaluated using the three classes of measures: graph (scalar), node, and link metrics. The link metrics have rarely been used to evaluate topology generators, but are shown to provide a more detailed and accurate analysis of the graph characteristics than traditionally used graph metrics.

A. *Graph Metrics*

Topology generators and their evolutionary properties have primarily been evaluated using only graph (scalar) metrics. However, graph metrics are the weakest since the space of graphs preserving a metric is considerably larger than node or link metrics (*i.e.,* graphs that have intrinsically different connectivity patterns will appear similar using this family of

TABLE III
Characteristics of recent RouteViews BGP topologies

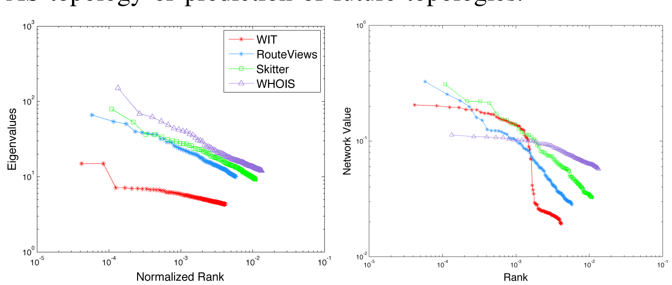
Year	$\|\mathbf{V}\|$	$\|\mathbf{E}\|$	k	r	$\bar{c}$	$\bar{d}$	$\bar{\eta}$
2004	19075	41646	4.36	-0.200	0.268	3.744	0.023
2005	21564	45459	4.21	-0.196	0.255	3.799	0.018
2006	24553	53897	4.39	-0.194	0.250	3.817	0.014
2007	27378	57428	4.19	-0.193	0.222	3.836	0.011
2008	30635	64113	4.18	-0.200	0.234	3.862	0.009
2009	33649	73330	4.35	-0.205	0.223	3.877	0.007
2010	36754	78429	4.26	-0.186	0.209	3.917	0.006
2011	38772	91470	4.71	-0.183	0.265	3.845	0.006

metrics). Hence, the graph metrics only provide basic insights into the graph structure which can sometimes be misleading. Nevertheless, we apply a few of these metrics to analyze the objectives of topology generators and the evolutionary characteristics of the Internet.

Table III lists the topology characteristics from the RouteViews dataset. We find that the average distance slightly increases and the average clustering decreases in the last few years as shown in Fig. 1, which is in contrast to previous observations of median distance [34], [35]. One possible explanation is that the tier 1-2 ISPs are merging leading to a dense network core, while tier 3-4 ISPs (*e.g.,* content providers) are expanding [13]. These recent changes in the Internet since 2007–2008 [23] impact models such as WIT that directly rely on preserving the distance and clustering coefficient over time. We further investigate these results by considering other distance variants such as the normalized decay centrality. This measure ($\delta = 0.5$) experiences a drop from the year 2004 to 2007, but stabilizes towards the more recent past ($2008 - 2011$). The decay centrality measure is highly sensitive to the changes in path length, and again we see a change occurring around 2007–2008.

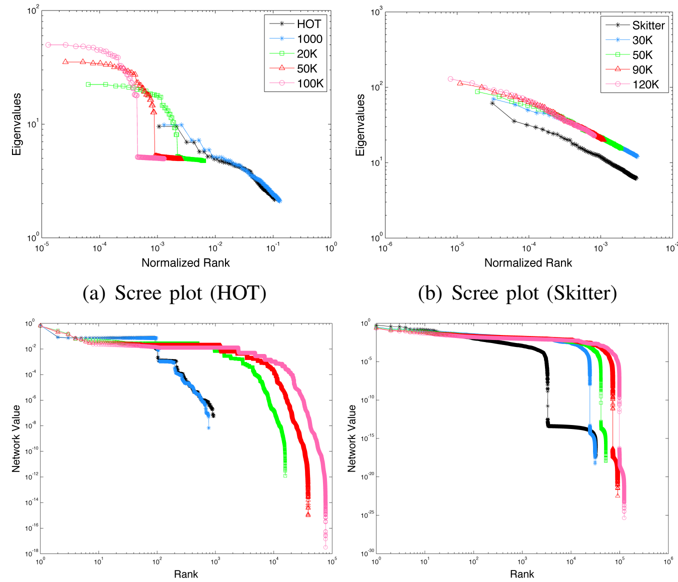
Orbis. We evaluate the two main objectives of Orbis using graph metrics. The Orbis topology generator attempts to preserve "any arbitrary" set of metrics as the size of the graph increases [25]. However, we find that as the number of nodes increases, the metrics from the generated topologies become increasingly uncorrelated (graph metrics deviate from the expected). This is most striking when applying Orbis on the HOT, WHOIS, and Skitter topologies (removed for brevity). Further, we find that almost all scalar metrics are increasingly inaccurate as we extrapolate further into the future. However, for generating topologies of the same size, we find that Orbis is able to preserve many of the graph metrics with reasonable accuracy.

We observe Orbis to have two intrinsic problems. First, the rescaling algorithm produces topologies of larger sizes that do not reflect the actual Internet evolution. Second, the rescaling algorithm based on interpolating the distribution introduces errors and noise. Therefore, Orbis is more appropriate for modeling static topologies, as opposed to the evolution of the AS topology or prediction of future topologies.



(a) Scree plot (b) Network values

Fig. 2. The WIT topology is compared to three different AS topologies (RouteViews, Skitter, and WHOIS) using two node metrics (Scree and top 100 network values). In all cases, WIT is very different. The topologies analyzed above were from March 2004, but similar results are obtained using topologies at different times.



(a) Scree plot (HOT) (b) Scree plot (Skitter)

(c) Network value (HOT) (d) Network value (Skitter)

Fig. 3. Scree plot and network values of the rescaled topologies generated using Orbis are compared to the initial HOT and Skitter (2004) topologies. As the number of nodes increases, the generated topologies (and metrics) become increasingly uncorrelated.

WIT. The WIT model was designed and evaluated under the assumption that the clustering coefficient and distance related measures are time-invariant [35], although the Internet is changing [23], [13]. We find that WIT does not accurately preserve the scalar metrics over time. The individual plots are omitted for brevity, but this point is clearly shown later in Fig. 8.

B. Node Metrics

While the evolutionary characteristics of the topology generators were previously shown to differ from the most recent evolution, we are interested in knowing why these generators fail (*i.e.,* use the metrics for forensic analysis), and understanding what important network characteristics are preserved, to what level, and at what evaluation granularity (graph/scalar, node, or link metrics). Following the multi-level framework, we analyze the topologies and generators in more detail using node metrics.

WIT. Fig. 2 plots the largest network values and singular-values of WIT and compares these with the corresponding benchmark topologies. Both significantly deviate from the others (the values are shifted and are of a different shape), even compared to RouteViews (used in the design and validation of WIT).

However, the metrics indicate several interesting insights into the topological features of the generators. First, the eigenvalues of the WIT topology in Fig. 2(a) are shifted downward, rotated, and are of a different shape than those found in the Internet.

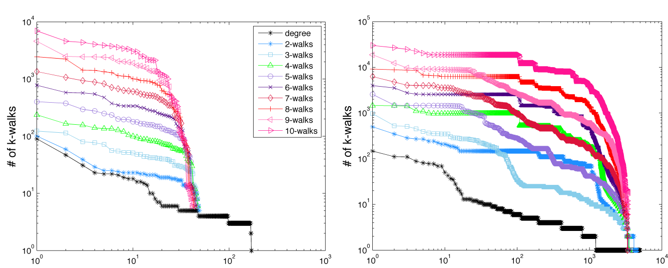
Besides WIT's network core represented by the first few large eigenvalues, the node clusters (approx. set of eigenvectors) and their corresponding weights given by the eigenvalues are more evenly distributed (or more linear) than the benchmark topologies. This indicates that topologies produced using WIT form more clusters than what is found in the Internet, and each of the clusters represents significantly different patterns of connectivity. The Internet AS topologies are more compressed (less unique patterns of connectivity), whereas the WIT topology appears more random.

Fig. 2(b) indicates a "phase transition" (backwards S shape) in the WIT topology that is not present in the Internet topologies. The network values found in WIT indicate two distinct clusters (or communities) of nodes which have very different structural properties. This could indicate structural characteristics of the network core. Nevertheless, these differences indicate the inaccuracy of one of the key assumptions used in the WIT model, when compared to the true connectivity, structure, and clusters found in the Internet.

Orbis. We compare Orbis with only two benchmark topologies for brevity. We find that at the static-level Orbis preserves the HOT topology scree plot shown in Fig. 3(a). However, as we model the evolution of the HOT topology using Orbis, the topologies become increasingly different. Similar conclusions were drawn from other benchmark topologies. The above interpretation for WIT can be applied in a similar fashion to Orbis.

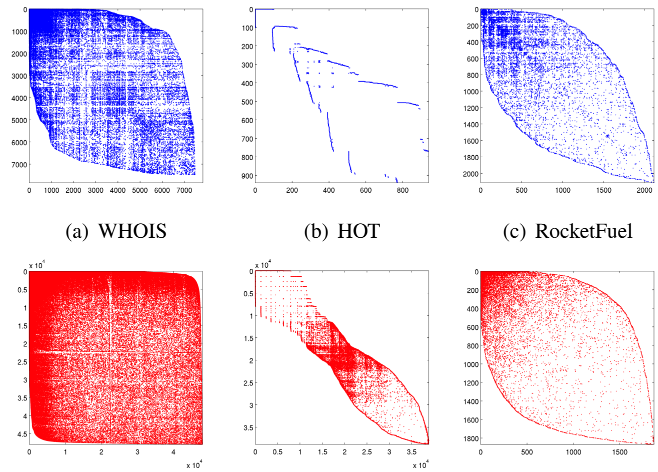
Finally, we analyze Orbis using k-walks. The motivation for the k-walks class of metrics arises from its intuitive definition with respect to local and global characteristics. For instance, $k = 1$ gives the degree of a node whereas $k = \infty$ gives the principal eigenvector.

We measured the ratio of walks using $k = \{1, 2, ..., 10\}$ for each topology. Fig. 4 illustrates the difference between the number of k-walks of various lengths. The measure of k-walks from the HOT topology depicted in Fig. 4(a) are *k-separated* (the number of structures is explicitly ordered according to k). In the case of Orbis shown in Fig. 4(b), the expected ordering clearly does not hold. This indicates a key problem with the Orbis-generated topology: several nodes are *disconnected*. The k-walk metric is useful not only for measuring and bounding the similarity of two graphs with respect to k, but also for forensic analysis of expected characteristics of generators. Results with other topologies and generators have been omitted for brevity.



(a) HOT (b) Orbis

Fig. 4. The ratios of *k-walks* from the HOT and the corresponding Orbis topology are analyzed using walks of length $\{1, ..., 10\}$. The behavior of the estimated topology (from Orbis) at various walk lengths deviates from the benchmark topology.



(a) WHOIS (b) HOT (c) RocketFuel

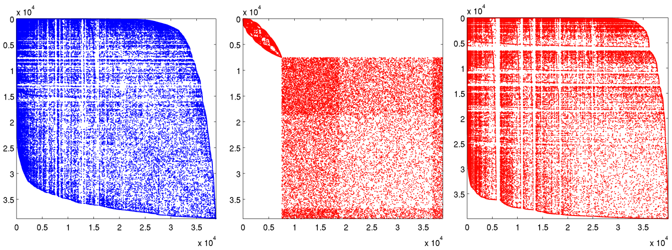
(d) Orbis (WHOIS) (e) Orbis (HOT) (f) Orbis (RocketFuel)

Fig. 5. Evaluating the connectivity and clustering of the generated topologies. The x-axis and y-axis of the above plots represent nodes of the adjacency matrices which are ordered by the principal singular-vector. **(a)**-**(c)** visualize the adjacency matrices for the benchmark topologies. The adjacency matrices for the corresponding generated topologies from Orbis are shown in **(d)**-**(f)**. The clustering properties (block structures) are found to be different in all topologies. For instance, **(c)** clearly shows signs of clusters (blocks/perpendicular lines), but the corresponding Orbis topology in **(f)** has no signs of the clusters previously observed and is much smoother. The main reason is that Orbis randomly rewires the links without considering the *community structure*.

C. Link Metrics: Structural Evaluation and Visualizations

We first compare the link structures by ordering the nodes (and weighting links) using the principal singular-vector. This vector corresponds to the largest singular-value. We chose the singular-vector since it pertains to the most significant link structures and connectivity patterns of the topology. Most often, this represents the network core and its connectivity patterns. Ordering the nodes by their distance from the main direction (how well they fit with the most significant connectivity patterns and link structures) shown in Fig. 5 and 6 provides evidence of the formation of clusters or groups (shown as block structures). Additionally, the visualization shows important properties such as the shape of connectivity, density of the border region, and the density of the clusters/block structures. These indicate the connectivity and expansion of the network core, and can provide intuition about distance-related properties. Most importantly, the resulting visualizations provide signatures of the topology and the corresponding generator to which that topology belongs (*i.e.,* they clearly distinguish generators from one another).

Traditionally, two topologies using an arbitrary link measure are compared by aggregating over link weights or attempting to match weighted link structures between the topologies. However, we provide a potentially more accurate and complete comparison by visualizing the adjacency matrix, allowing for an easy and simple assessment of similarity. A topology is similar at the global, community, and local levels if we observe similar clusters (most evidently the network core), shape, density of the shapes in the border region, and other unique connectivity patterns. If we find these properties to be



(a) RouteViews 2011 (b) WIT 2011 (c) Orbis 2011

Fig. 6. Evaluating the connectivity and clustering of the generated topologies using the most recent Internet AS topology (see **(a)**). **(b)** The WIT topology exhibits very different characteristics from the other topologies and contains an obvious signature that distinguishes WIT topologies from the other Internet AS topologies or generators. The tightly-knit cluster formed at the top of the WIT visualizations barely connects to the rest of the network, which indicates an intrinsic problem with the generator. **(c)** The topologies generated from Orbis preserve the characteristics (clustering, link structure, and connectivity patterns) more accurately than WIT (closer to that of the RouteViews).

significantly different, then this implies the principal singular-vector – denoting the main direction and hence connectivity – must be different.

Fig. 5(a)-5(c) visualize the benchmark topologies. The corresponding Orbis visualizations are shown in Fig. 5(d)-5(f). The clustering properties (block structures) are found to be significantly different in all topologies. For instance, Fig. 5(c) clearly shows signs of clusters, but the corresponding Orbis topology in Fig. 5(f) has no signs of the clusters previously observed and is much smoother. The links are randomly rewired without considering the *community structure*. Similar observations are seen with the other topologies. Additionally, the link structure and connectivity patterns are significantly different for all topologies as clearly seen from the shape of the connectivity and border formed from the implicit ordering.

The topologies generated using WIT and Orbis over time and their visualizations are depicted in Fig. 6. The WIT topology exhibits significantly different characteristics from any of the topologies (shown in Fig. 5) and contains an obvious signature that distinguishes WIT topologies from Internet AS topologies or other generators. The tightly-knit cluster formed at the top of the WIT visualizations barely connects to the rest of the network, which indicates an intrinsic problem with the generator. We can also directly compare WIT with the Skitter and WHOIS topologies from Fig. 5.

The evolutionary characteristics of WIT and Orbis are now compared to the Internet AS at the link-metrics level, the finest evaluation granularity. We find that the topologies generated from Orbis over time preserve the evolutionary characteristics (clustering, link structure, and connectivity patterns) more accurately than WIT as shown in Fig. 6.

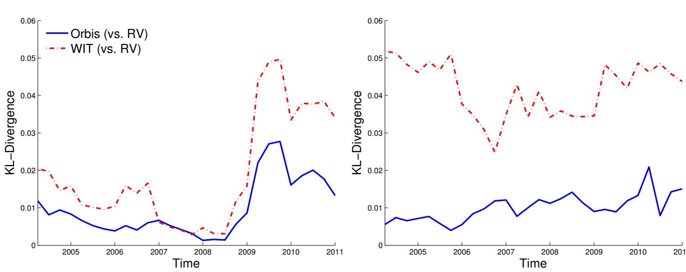
Perhaps the most striking visualization is that shown in Fig. 7. This approximates the topologies using only the most significant singular-vector determined by the largest singular-value. Other k-approximations can be generated using the first k singular-vectors. If $k = |V|$, then the result is the original topology, whereas different choices of k capture other network characteristics.

The visualization in Fig. 7 is of the network core and its weighted connectivity – links are created, suppressed, and assigned weights. The core of the Internet is larger and the connectivity is more highly weighted compared to Orbis and WIT. The link metrics allow for direct comparison of the complete topology using orderings and approximations that reveal the most significant local and global properties.

D. A Single Measure of Evaluation

We have demonstrated that Orbis and WIT fail to capture certain metrics from each of the three classes (graph, node, and link metrics). We now evaluate the generators by measuring the KL-divergence of their set of metrics over time, using the RouteViews data as ground-truth (true distribution). The KL-divergence of both the selected set of metrics and the learned set (using ReFex [18]) are computed across time. The results are shown in Fig. 8. The Orbis topologies are found to fit more closely with the RouteViews topologies (lower divergence from the true distribution). This is true for both the selected and learned set of metrics. The KL-divergence provides a single measure for evaluating topologies based on the combined set of metrics. Learning a set of metrics automatically based on a distribution of graphs allows generators to be evaluated systematically using graph features that are representative of the true distribution.

The learned graph metrics for RouteViews and the generators are visualized in Fig. 9 by considering the normalized weight of each across time. Interestingly, the 15th-19th features transition from having low weight to very high weight in the more recent Internet structure (captured better by Orbis than WIT). Nevertheless, the learned features from Orbis seemingly match the Internet AS better than WIT as shown more clearly in Fig. 8(b).

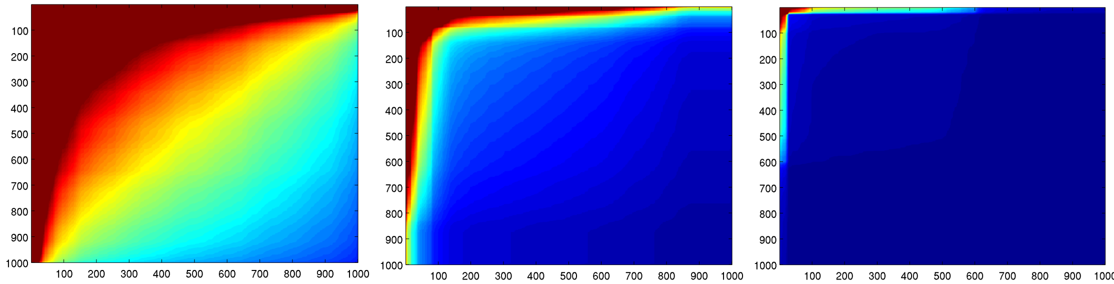


(a) Selected metrics (b) Learned metrics

Fig. 8. Topology generators are evaluated over time (2004-2011) by computing the KL-divergence of their combined set of metrics and the RouteViews data (used as ground-truth). We find that Orbis preserves the properties of the static topology with reasonable accuracy (used the 2004 RouteViews topology as input), but as this topology is rescaled to a larger size, the properties diverge more from the true distributions (shown by the increasing trend in **(a)** and **(b)**). The combined set of metrics from the WIT topologies over time do not match the Internet AS (if WIT tracked the properties perfectly, then the WIT curve would be an horizontal line on zero). In all cases, the Orbis topologies are shown to be more similar (diverge less) to RouteViews.

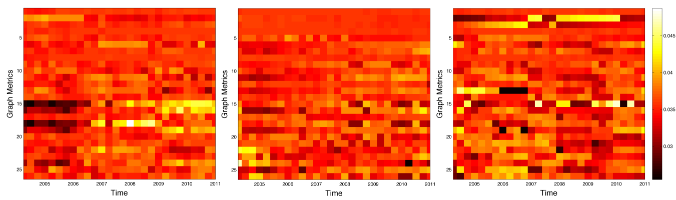
VI. Conclusions

In this paper, we have proposed a multi-level framework for understanding Internet topologies, their evolution, and for comparing topology generators. We used the framework



(a) RouteViews (b) Orbis (c) WIT

Fig. 7. RouteViews AS topology (2011) compared to the corresponding Orbis and WIT topologies. The ASes in each topology are first ranked according to their coordinate value on the principal singular-vector then the connectivity patterns (and the latent link weights/strengths) of the network core are estimated using the closest rank-k approximation (given by SVD). The resulting visualization is of the network core and the corresponding cluster properties.



(a) RouteViews (b) Orbis (c) WIT

Fig. 9. Comparison of the learned graph metrics across time (2004-2011) for each topology generator. The Orbis generator is shown to be more similar to the Internet while WIT appears more random. Interestingly, there is a significant evolutionary transition in the Internet captured by the 15-19th graph metrics.

to evaluate whether the recent generators Orbis and WIT preserved a wide-range of important network properties and compared their ability to preserve these characteristics as the network evolves. We identified a number of shortcomings of both generators. We also observed that recent Internet evolutionary characteristics significantly differ from trends assumed by many Internet topology generators.

Traditionally, topology generators have evaluated evolutionary properties using a few macro metrics which often lead to misleading conclusions. For this reason, we used a multi-level approach that leverages both macro metrics (graph) and micro metrics (node and link metrics) to more accurately compare topologies while capturing the important network characteristics warranted by researchers. Our results suggest that existing topology generators fail to accurately model the evolution of the Internet AS topology. More unexpectedly, we found that many generators fail to capture important static characteristics.

In general, we found the data-driven generators, *e.g.,* Orbis, to be more accurate than the generators based on a mechanism with no estimation (such as WIT). Data-driven generators preserve the properties of an input topology, but generate static topologies with low or no variance. However, for modeling the evolution of the Internet, the properties become significantly uncorrelated as the size increases. Conversely, the parametric generators fail to model the Internet evolution if any key assumption is violated or the assumed characteristics change [23]. Moreover, if their models lack parameter estimation, then selecting a set of reasonable parameters becomes extremely difficult in practice.

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