From Closing Triangles to Closing Higher-Order Motifs

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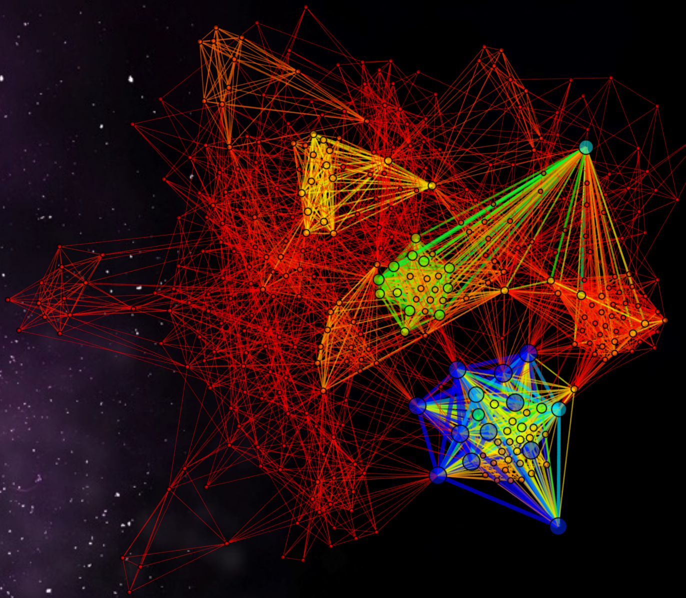
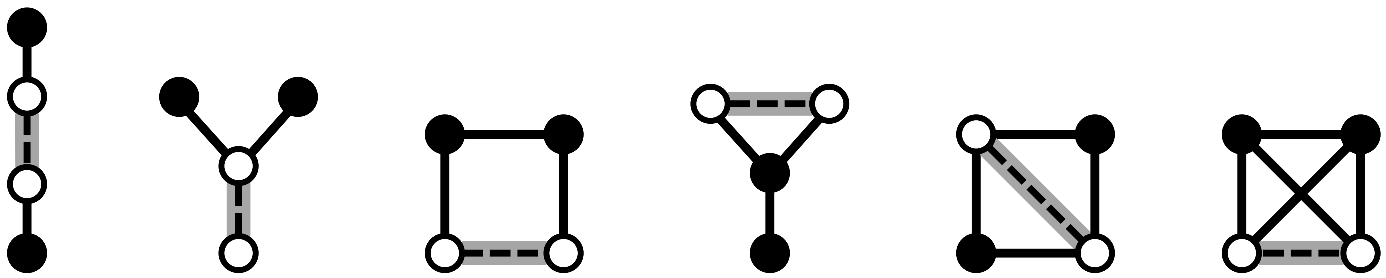


MAKE IT AN EXPERIENCE

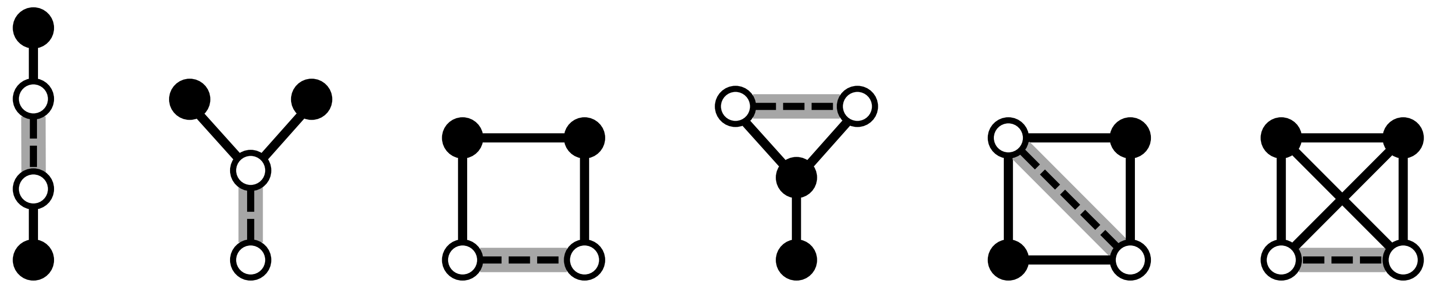
Higher-Order Motif Closures

- Proposed general notion of a motif closure that goes beyond simple triangle closures
- Introduce higher-order ranking and link prediction methods based on closing higher-order network motifs
- Demonstrate that these new motif closures often outperform triangle-based methods



Definition 1 (Motif Closure). *A node pair* (i, j) *is said to* close *a motif* H *iff adding an edge* (i, j) *to* E closes *an instance* $F \in I_{G'}(H)$ *of motif* H *where* $G' = (V, E \cup \{(i, j)\})$ *and* $I_{G'}(H)$ *is the set of unique instances of motif* H *in* G'.

Higher-Order Motif Closure Frequency

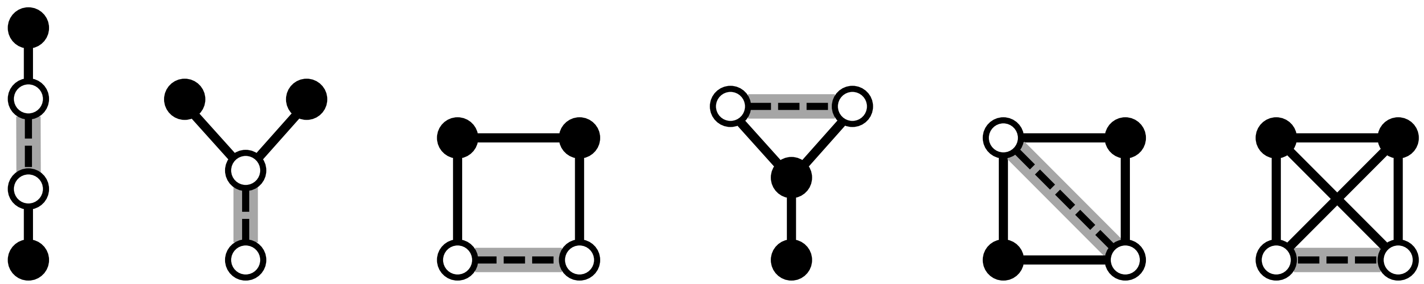


Definition 2 (Higher-Order Motif Closure Frequency). *Let $G' = (V, E')$ where $E' = E \cup \{(i, j)\}$ and let $I_{G'}(H)$ be the set of unique instances of motif H in G'. Then the frequency of closing a higher-order motif H between node i and j is:*

$$W_{ij} = \sum_{F \in I_{G'}(H)} \mathbb{I}\big(\{i, j\} \in E'(F)\big) \tag{1}$$

where W_{ij} is equal to the number of unique instances of H that contain nodes $\{i, j\} \subset V(G')$ as an edge.

Higher-Order Motif Closure Frequency



Algorithm 1 Higher-Order Motif Closures

Input: a graph $G = (V, E)$, node pair (i, j), and network motif/graphlet H
Output: the frequency W_{ij} of motif closures of H for nodes i and j

1 Set $E' \leftarrow E \cup \{(i, j)\}$ and $G' = (V, E')$
2 Use fast algorithm [1, 4] to compute W_{ij} = # of occurrences of motif H between node i and j in G'

Experiments

The experiments investigate the following key questions:

- **Q1.** Do other motif closures perform better than triangle closure & its variants for some graphs?
- **Q2.** Does the "best" motif closure depend highly on the underlying network and its structural properties or is there one motif closure that always outperforms the others?

Experimental Setup

- Hold-out 10% of the observed node pairs uniformly at random
- Randomly sample the same number of negative node pairs
- Use the methods to obtain a ranking of the node pairs
- Repeat this 10 times and average the result

Results

Mean average precision (MAP) results for ranking methods based on closing higher-order motifs.

	bn-mouse	*bio-DM-LC*	*bio-CE-HT*	*bio-DM-HT*	*ia-reality*	*web-polblogs*	*biogrid-worm*	*biogrid-plant*	*biogrid-yeast*	*email-dnc-corec.*	*soc-advogato*	*econ-wm1*
4-path	0.829	0.687	0.607	0.594	0.649	0.778	0.865	0.729	0.893	0.873	0.914	0.788
4-star	0.880	0.787	0.595	0.696	**0.922**	0.814	0.895	0.861	0.840	0.813	0.889	0.688
4-cycle	**0.881**	**0.958**	**0.651**	**0.926**	0.827	0.885	**0.908**	**0.935**	0.927	0.957	0.930	0.900
4-tailed-triangle	0.804	0.612	0.570	0.752	0.773	0.663	0.773	0.681	0.689	0.779	0.600	0.496
4-chordal-cycle	0.801	0.837	0.598	0.842	0.312	**0.966**	0.840	0.854	**0.977**	0.996	**0.986**	0.947
4-clique	0.804	0.838	0.595	0.843	0.293	0.963	0.842	0.847	0.972	**0.997**	**0.986**	**0.965**
CN	0.705	0.872	0.613	0.839	0.422	0.814	0.833	0.897	0.839	0.960	0.949	0.852
Jaccard Sim.	0.705	0.873	0.618	0.841	0.537	0.933	0.853	0.918	0.955	**0.997**	0.973	0.918
Adamic/Adar	0.705	0.883	0.621	0.842	0.549	0.940	0.856	0.920	0.959	**0.997**	0.976	0.919

Result 1. Higher-order motif closures can outperform triangle closure (common neighbors) and other methods based on it

Results

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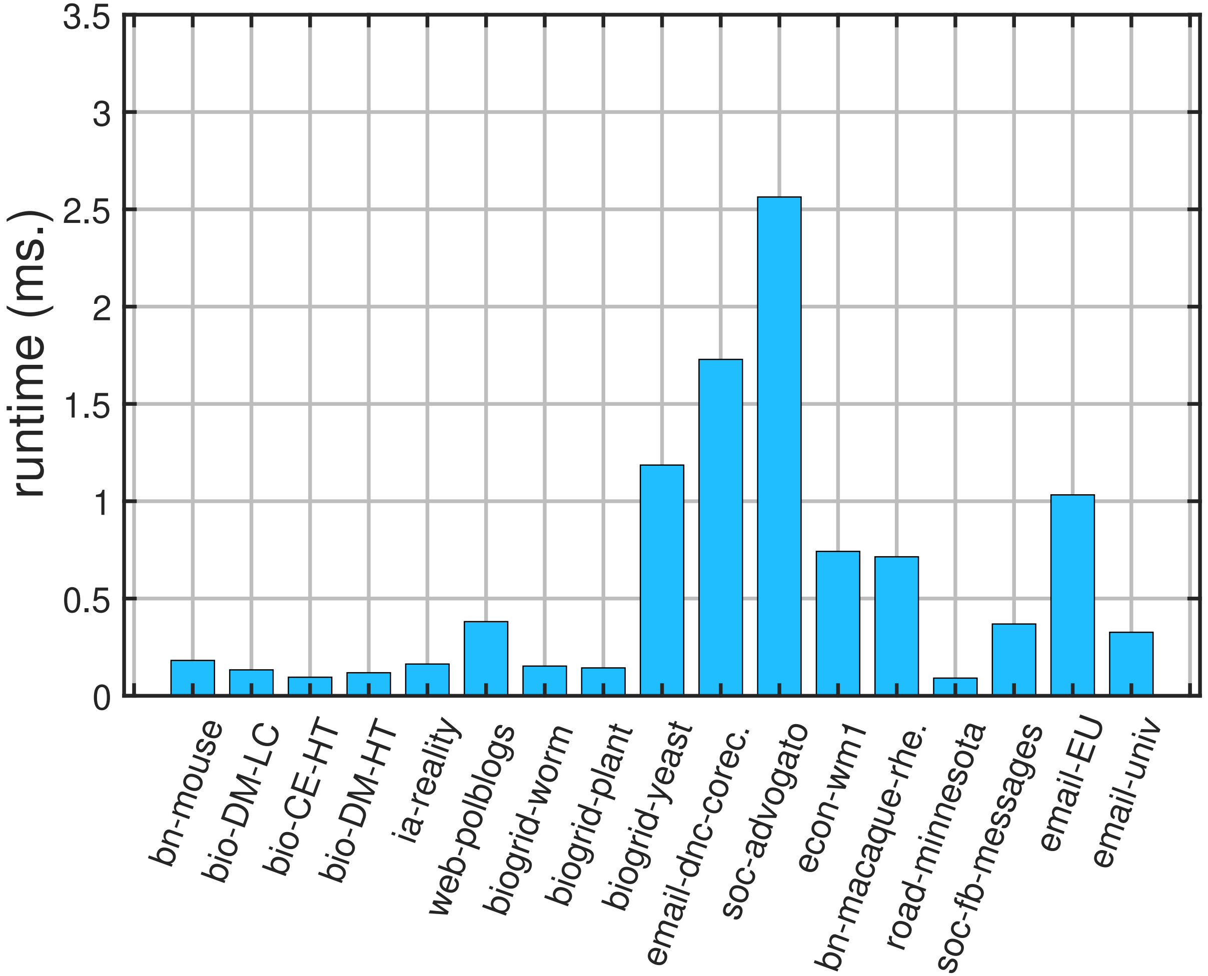
Result 2. The best motif closure depends highly on the structural characteristics of the graph and its domain (biological vs. social network) as shown in the above Table.

Results

- Average runtime in milliseconds to compute all {3, 4}-node motif closures for each node pair.
- The runtime includes the baselines since they require 3-node motifs.



Result 3. For any 4-node motif H, counting the number of motif closures W_{ij} that would arise if an edge between i and j was added to G is fast taking less than a millisecond on average across all graphs

Summary of Contributions

- Proposed General Notion of "Motif Closure"
  - Moves beyond simple triangle closures

- Motif closures are often more predictive than triangle closures

- Important Findings & Implications of Results
  - Need to consider other motif closures (besides triangle closures)
  - Best motif closure depends highly on the network structure and processes governing it
  - Existing supervised methods can benefit new motif closures (by leveraging full spectrum, most dense to least)

Thanks for listening!

Appendix